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Notes on a generalized *abc*-conjecture over function fields

Pei-Chu Hu & Chung-Chun Yang*

Abstract

In this note, we weaken a condition in the generalized *abc*-conjecture proposed by us in a previous paper, and prove its analogue for non-Archimedean entire functions, as well as a generalized Mason's theorem for polynomials.

1 Introduction

In all the paper, κ will denote an algebraically closed field of characteristic zero.

Let a be a non-zero integer. Then

$$|a| = p_1^{i_1} \cdots p_n^{i_n}$$

holds for distinct primes p_1, \dots, p_n . For a positive integer k , define

$$r_k(a) = \prod_{\nu=1}^n p_\nu^{\min\{i_\nu, k\}}.$$

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Conjecture 1.1 ([6],[10],[11]) *Let $a_j (j = 0, \dots, k)$ be nonzero integers such that the greatest common factor of a_0, \dots, a_k is 1,*

$$a_1 + \dots + a_k = a_0, \quad (1)$$

and no proper subsum of (1) is equal to 0. Then for $\varepsilon > 0$, there exists a number $C(k, \varepsilon)$ such that $\max_{0 \leq j \leq k} \{|a_j|\} \leq C(k, \varepsilon) \left(\prod_{i=0}^k r_{k-1}(a_i) \right)^{1+\varepsilon}$, $\max_{0 \leq j \leq k} \{|a_j|\} \leq C(k, \varepsilon) r_{\frac{k(k-1)}{2}}(a_0 \cdots a_k)^{1+\varepsilon}$.

If $k = 2$, this corresponds to the well known *abc*-conjecture which also is a consequence of the Vojta's Conjecture (see Vojta [19]). Some special cases of Conjecture 1.1 were given in [8] and [9]. In this note, we will prove the analogue of Conjecture 1.1 for entire functions defined over non-Archimedean fields:

Theorem 1.1 *Let κ be complete for a non-trivial non-Archimedean absolute value $|\cdot|$. Let $f_j (j = 0, \dots, k)$ be entire functions on κ such that f_0, \dots, f_k have no common zeros, $f_j (j = 1, \dots, k)$ be linearly independent on κ and*

$$f_1 + \dots + f_k = f_0. \quad (2)$$

Then the Nevanlinna functions $T(r, f_j)$ and $N_k(r, f_j)$ satisfy

$$\begin{aligned} \max_{0 \leq j \leq k} \{T(r, f_j)\} &\leq \sum_{i=0}^k N_{k-1}\left(r, \frac{1}{f_i}\right) - \frac{k(k-1)}{2} \log r + O(1) \\ \max_{0 \leq j \leq k} \{T(r, f_j)\} &\leq N_{\frac{k(k-1)}{2}}\left(r, \frac{1}{f_0 \cdots f_k}\right) - \frac{k(k-1)}{2} \log r + O(1). \end{aligned}$$

For the meaning of the above notations, we refer the reader to § 2. Under a stronger condition that f_0, f_j have no common zeros for $j = 1, \dots, k$, some special cases of Theorem 1.1 were given in [7], [9]. If f is a polynomial, it is easy to show

$$\deg(f) = \lim_{r \rightarrow \infty} \frac{T(r, f)}{\log r}, \quad r_k(f) := \lim_{r \rightarrow \infty} n_k\left(r, \frac{1}{f}\right) = \lim_{r \rightarrow \infty} \frac{N_k\left(r, \frac{1}{f}\right)}{\log r}.$$

As a direct consequence of Theorem 1.1, we obtain

Theorem 1.2 *Let $f_j (j = 0, \dots, k)$ be polynomials on κ such that f_0, \dots, f_k have no common zeros, $f_j (j = 1, \dots, k)$ be linearly independent on κ and*

$$f_1 + \dots + f_k = f_0. \quad (3)$$

Then $\max_{0 \leq j \leq k} \{\deg(f_j)\} \leq \sum_{i=0}^k r_{k-1}(f_i) - \frac{k(k-1)}{2}$, $\max_{0 \leq j \leq k} \{\deg(f_j)\} \leq r_{\frac{k(k-1)}{2}}(f_0 \cdots f_k) - \frac{k(k-1)}{2}$.

When $k = 2$, it reduces to a Mason's theorem (see [12], [13], [14], [15] and [18]) which has been generalized recently to fields of any characteristic p by Boutabaa and Escassut [2]. If $k \geq 2$, the following example

$$f_0(z) = (z + 1)^{k-1}, \quad f_{i+1}(z) = k - iz^i \quad (i = 0, \dots, k - 1),$$

which obviously satisfy the conditions in Theorem 1.2, shows that the inequalities in the theorem, in fact, become equality for this example. Under the stronger assumption that f_j, f_0 have no common zeros for $j = 1, \dots, k$, Theorem 1.2 was obtained by Hu-Yang [9]. For any positive integer k and any polynomial f on κ , note that

$$r_k(f) \leq kr_1(f).$$

Theorem 1.2 yields immediately the following:

Theorem 1.3 *Let f_1, f_2, \dots, f_k ($k \geq 2$) be linearly independent polynomials in κ . Put $f_0 = f_1 + f_2 + \dots + f_k$ and assume that f_0, \dots, f_k have no common zeros. Then the following inequalities $\max_{0 \leq j \leq k} \{\deg(f_j)\} \leq (k - 1) \sum_{i=0}^k r_1(f_i) - \frac{k(k-1)}{2}$, $\max_{0 \leq j \leq k} \{\deg(f_j)\} \leq \frac{k(k-1)}{2} (r_1(f_0 \cdots f_k) - 1)$, hold.*

The inequality (1.3) was obtained independently by J. F. Voloch [20], W. D. Brownawell and D. Masser [4]. Earlier R. C. Mason [16] derived this estimate with $\frac{1}{2}k(k-1)$ replaced by 4^{k-1} . J. Browkin and J. Brzeziński [3] conjectured that the value $\frac{1}{2}k(k-1)$ in (1.3) would be replaced by $2k-3$. If the restriction on the linear independence of polynomials f_1, \dots, f_k is removed, we have

Theorem 1.4 *For fixed integer $k \geq 1$, let f_j ($j = 0, \dots, k$) be non-zero polynomials on κ such that $f_1 + \dots + f_k = f_0$. Assume also that not all the f_j are constants, and the f_j are pairwise relatively prime. Then*

$$\max_{0 \leq j \leq k} \{\deg(f_j)\} \leq (d - 1) (r_1(f_0 \cdots f_k) - 1), \quad (4)$$

where d is the dimension of the vector space spanned by the f_i over κ .

As an application of Theorem 1.2, we can derive the following:

Theorem 1.5 *Given polynomials f_1, f_2, \dots, f_k ($k \geq 2$) in κ and positive integers l_j ($1 \leq j \leq k$) such that*

- (a) $f_1^{l_1}, f_2^{l_2}, \dots, f_k^{l_k}$ are linearly independent over κ ;
 (b) $f_0, f_1^{l_1}, f_2^{l_2}, \dots, f_k^{l_k}$ have no common zeros, where

$$f_0 = \sum_{j=1}^k f_j^{l_j}.$$

Then the following inequality

$$\left\{ 1 - \sum_{j=1}^k \frac{k-1}{l_j} \right\} \max_{1 \leq j \leq k} \deg(f_j^{l_j}) \leq r_{k-1}(f_0) - \frac{k(k-1)}{2} \quad (5)$$

holds.

Obviously, the inequality (5) implies

$$\left\{ 1 - \sum_{j=1}^k \frac{k-1}{l_j} \right\} \max_{1 \leq j \leq k} \deg(f_j^{l_j}) \leq \deg\left(\sum_{j=1}^k f_j^{l_j}\right) - \frac{k(k-1)}{2}. \quad (6)$$

For the case

$$k = 2, \quad l_1 = 2, \quad l_2 = 3, \quad (7)$$

and

$$f_1 = f, \quad f_2 = -g, \quad (8)$$

the inequality (6) yields

$$\frac{1}{2} \deg(g) \leq \deg(f^2 - g^3) - 1, \quad (9)$$

which was proved for complex case by Davenport [5]. In fact, Davenport proved that (9) is true as long as $f^2 - g^3 \neq 0$ (also see [1],[18]).

2 Basic facts

Let κ be an algebraically closed field of characteristic zero, complete for a non-trivial non-Archimedean absolute value $|\cdot|$. Define

$$\kappa[0; r] = \{z \in \kappa \mid |z| \leq r\}.$$

Let $\mathcal{A}(\kappa)$ be the set of entire functions on κ . Then each $f \in \mathcal{A}(\kappa)$ can be given by a power series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n, \quad (a_n \in \kappa), \quad (10)$$

such that for any $z \in \kappa$, one has $|a_n z^n| \rightarrow 0$ as $n \rightarrow \infty$. Define the *maximum term*:

$$\mu(r, f) = \max_{n \geq 0} |a_n| r^n$$

with the associated the *central index*:

$$n\left(r, \frac{1}{f}\right) = \max_{n \geq 0} \{n \mid |a_n| r^n = \mu(r, f)\}.$$

Then $n\left(r, \frac{1}{f}\right)$ just is the *counting function* of zeros of f , which denotes the number of zeros (counting multiplicity) of f with absolute value $\leq r$. Fix a real ρ_0 with $\rho_0 > 0$. Define the *valence function* of zeros of f by

$$N\left(r, \frac{1}{f}\right) = \int_{\rho_0}^r \frac{n(t, \frac{1}{f})}{t} dt \quad (r > \rho_0). \quad (11)$$

The field of fractions of $\mathcal{A}(\kappa)$ will be denoted by $\mathcal{M}(\kappa)$. An element f in the set $\mathcal{M}(\kappa)$ will be called a *meromorphic function* on κ . Take $f \in \mathcal{M}(\kappa)$. Since greatest common divisors of any two elements in $\mathcal{A}(\kappa)$ exist, then there are $g, h \in \mathcal{A}(\kappa)$ with $f = \frac{g}{h}$ such that g and h have no any common zeros in the ring $\mathcal{A}(\kappa)$. We can uniquely extend μ to a meromorphic function $f = \frac{g}{h}$ by defining

$$\mu(r, f) = \frac{\mu(r, g)}{\mu(r, h)} \quad (0 \leq r < \infty).$$

Then the following *Jensen formula*

$$N\left(r, \frac{1}{f}\right) - N(r, f) = \log \mu(r, f) - \log \mu(\rho_0, f) \quad (12)$$

holds, where

$$N(r, f) = N\left(r, \frac{1}{h}\right).$$

Note that

$$\mu(r, f_1 f_2) = \mu(r, f_1) \mu(r, f_2), \quad f_1, f_2 \in \mathcal{M}(\kappa).$$

Thus the Jensen formula implies

$$N\left(r, \frac{1}{f_1 f_2}\right) - N(r, f_1 f_2) = N\left(r, \frac{1}{f_1}\right) + N\left(r, \frac{1}{f_2}\right) - N(r, f_1) - N(r, f_2). \quad (13)$$

Define the *compensation function* by

$$m(r, f) = \max\{0, \log \mu(r, f)\}.$$

As usual, we define the *characteristic function*:

$$T(r, f) = m(r, f) + N(r, f) \quad (\rho_0 < r < \infty).$$

Then the following formula (see [8])

$$T(r, f) = \max\left\{N\left(r, \frac{1}{f-a}\right), N\left(r, \frac{1}{f-b}\right)\right\} + O(1) \quad (14)$$

holds for any two distinct elements $a, b \in \kappa \cup \{\infty\}$. In particular, if f is a non-constant entire function in κ , then

$$N\left(r, \frac{1}{f-a}\right) = T(r, f) + O(1) \quad (15)$$

for all $a \in \kappa$.

We also denote the number of distinct zeros of $f-a$ on $\kappa[0; r]$ by $\bar{n}(r, \frac{1}{f-a})$ and define

$$\bar{N}\left(r, \frac{1}{f-a}\right) = \int_{\rho_0}^r \frac{\bar{n}(t, \frac{1}{f-a})}{t} dt \quad (r > \rho_0).$$

Let $n_k(r, \frac{1}{f-a})$ denote the the number of zeros of $f-a$ on $\kappa[0; r]$, where a zero of $f-a$ with multiplicity m will be counted as $\min\{m, k\}$ in $n_k(r, \frac{1}{f-a})$. Write

$$N_k\left(r, \frac{1}{f-a}\right) = \int_{\rho_0}^r \frac{n_k(t, \frac{1}{f-a})}{t} dt \quad (r > \rho_0).$$

The following result is a non-Archimedean analogue of a result of Nevanlinna[17]:

Lemma 2.1 ([8], [9]) *Let $f_j (j = 1, \dots, k)$ be linearly independent meromorphic functions on κ such that*

$$f_1 + \dots + f_k = 1. \quad (16)$$

Then

$$\begin{aligned} T(r, f_j) &< \sum_{i=1}^k N\left(r, \frac{1}{f_i}\right) - \sum_{i \neq j} N(r, f_i) + N(r, \mathbf{W}) \\ &\quad - N\left(r, \frac{1}{\mathbf{W}}\right) - \frac{k(k-1)}{2} \log r + O(1), \quad 1 \leq j \leq k \quad (17) \end{aligned}$$

where \mathbf{W} is the Wronskian of f_1, \dots, f_k .

3 Proof of the main theorems

Proof of Theorem 1.1. Applying Lemma 2.1 to $f_1/f_0, \dots, f_k/f_0$, we obtain

$$\begin{aligned} T\left(r, \frac{f_j}{f_0}\right) &< \sum_{i=1}^k N\left(r, \frac{f_0}{f_i}\right) - \sum_{i \neq j} N\left(r, \frac{f_i}{f_0}\right) + N(r, \mathbf{W}) \\ &\quad - N\left(r, \frac{1}{\mathbf{W}}\right) - \frac{k(k-1)}{2} \log r + O(1), \quad 1 \leq j \leq k, \quad (18) \end{aligned}$$

where $\mathbf{W} = \mathbf{W}(f_1/f_0, \dots, f_k/f_0)$ is the Wronskian of $f_1/f_0, \dots, f_k/f_0$. Note that

$$\mathbf{W} = \mathbf{W}\left(\frac{f_1}{f_0}, \dots, \frac{f_k}{f_0}\right) = \frac{\mathbf{W}_1}{f_0^k},$$

where $\mathbf{W}_1 = \mathbf{W}(f_1, \dots, f_k)$ is the Wronskian of f_1, \dots, f_k . By the formula (13), we obtain easily $N\left(r, \frac{f_0}{f_i}\right) - N\left(r, \frac{f_i}{f_0}\right) = N\left(r, \frac{1}{f_i}\right) - N\left(r, \frac{1}{f_0}\right)$,

$N(r, \mathbf{W}) - N\left(r, \frac{1}{\mathbf{W}}\right) = kN\left(r, \frac{1}{f_0}\right) - N\left(r, \frac{1}{\mathbf{W}_1}\right)$, and hence, for $1 \leq j \leq n$, we obtain

$$\begin{aligned} T\left(r, \frac{f_j}{f_0}\right) &\leq \sum_{i=0}^n N\left(r, \frac{1}{f_i}\right) - N\left(r, \frac{1}{\mathbf{W}_1}\right) + N\left(r, \frac{f_0}{f_j}\right) \\ &\quad - N\left(r, \frac{1}{f_j}\right) - \frac{k(k-1)}{2} \log r + O(1). \quad (19) \end{aligned}$$

Obviously, for each $j = 1, \dots, k$, we can choose entire functions h_j, \tilde{f}_{0j} and \tilde{f}_j such that \tilde{f}_{0j} and \tilde{f}_j have no common zeros, and

$$f_0 = h_j \tilde{f}_{0j}, \quad f_j = h_j \tilde{f}_j.$$

By simple observation, we find

$$N\left(r, \frac{1}{f_j}\right) - N\left(r, \frac{f_0}{f_j}\right) = N\left(r, \frac{1}{h_j}\right). \quad (20)$$

Noting that, by (14) and (15),

$$\begin{aligned} T\left(r, \frac{f_j}{f_0}\right) + N\left(r, \frac{1}{h_j}\right) &= \max\left\{N\left(r, \frac{1}{f_0}\right), N\left(r, \frac{1}{f_j}\right)\right\} + O(1) \\ &= \max\{T(r, f_0), T(r, f_j)\} + O(1). \end{aligned} \quad (21)$$

Thus Theorem 1.1 follows from (19), (21) and the following estimates

$$\sum_{i=0}^k \mu_{f_i}^0 - \mu_{W_1}^0 \leq \sum_{i=0}^k \mu_{f_i, k-1}^0, \quad (22)$$

$$\sum_{i=0}^k \mu_{f_i}^0 - \mu_{W_1}^0 \leq \mu_{f_0 \dots f_k, \frac{k(k-1)}{2}}^0, \quad (23)$$

where μ_f^a is the a -valued multiplicity of an element $f \in \mathcal{M}(\kappa)$, and

$$\mu_{f, k}^a(z) = \min\{\mu_f^a(z), k\}.$$

Take $z_0 \in \kappa$. Then $\mu_{f_s}^0(z_0) = 0$ for some $s \in \{0, \dots, n\}$ since f_0, \dots, f_k have no common zeros. Note that, by the identity (2),

$$W_1 = W(f_1, \dots, f_{s-1}, f_0, f_{s+1}, \dots, f_n).$$

Obviously we have

$$\mu_{f_i^{(j)}}^0(z_0) \geq \mu_{f_i}^0(z_0) - \mu_{f_i, j}^0(z_0) \geq \mu_{f_i}^0(z_0) - \mu_{f_i, k-1}^0(z_0), \quad i \neq s, 1 \leq j \leq k-1,$$

and, hence,

$$\mu_{W_1}^0(z_0) \geq \sum_{i \neq s} \{\mu_{f_i}^0(z_0) - \mu_{f_i, k-1}^0(z_0)\},$$

that is, $\sum_{i=0}^k \mu_{f_i}^0(z_0) - \mu_{W_1}^0(z_0) = \sum_{i \neq s} \mu_{f_i}^0(z_0) - \mu_{W_1}^0(z_0) \leq \sum_{i \neq s} \mu_{f_i, k-1}^0(z_0) = \sum_{i=0}^k \mu_{f_i, k-1}^0(z_0)$. The inequality (23) can be obtained similarly by comparing the multiplicities of zeros of $f_0 \cdots f_k$ and W_1 . Then Theorem 1.1 follows from (19), (21), (22) and (23).

Proof of Theorem 1.4. In the sequel, we will use the notation

$$f_i \equiv 0\{f_{i_1}, \dots, f_{i_{s_i}}\} \quad (24)$$

to denote that $\{i_1, \dots, i_{s_i}\} \subset \{0, 1, \dots, k\} - \{i\}$ are distinct, $f_{i_1}, \dots, f_{i_{s_i}}$ linearly independent, and

$$f_i = \sum_{\alpha=1}^{s_i} c_\alpha f_{i_\alpha}, \quad c_\alpha \in \kappa - \{0\} \quad (1 \leq \alpha \leq s_i).$$

We proceed the proof of Theorem 1.4 by induction on k . For $k = 1$ it is obviously true since if $f_0 = f_1$, f_0 and f_1 relatively prime, then they both are constants. Assume the theorem is true for all cases k' with $2 \leq k' < k$, and consider that of $k+1$ polynomials. By the assumptions in Theorem 1.4, at least two of the f_i are non-constant. Note that if two of the f_i are constants, then we may either eliminate them if their sum is zero or replace them by their sum when it is not zero. Then the inductive hypothesis could be applied to yield the desired result. Thus we may assume that at most one of the f_i is a constant. For each $i \in \{0, 1, \dots, k\}$, it is easy to show that

$$f_i \equiv 0\{f_{i_1}, \dots, f_{i_{s_i}}\}$$

for some i_1, \dots, i_{s_i} . Obviously, $d \geq s_i \geq 2$ and $f_{i_1}, \dots, f_{i_{s_i}}$ have no common zeros since the f_j are pairwise relatively prime. So by Theorem 1.3, we have

$$\max_{0 \leq \alpha \leq s_i} \{\deg(f_{i_\alpha})\} \leq (s_i - 1) \sum_{\alpha=0}^{s_i} r_1(f_{i_\alpha}) - \frac{s_i(s_i - 1)}{2}, \quad (25)$$

where $i_0 = i$. Therefore, we obtain

$$\begin{aligned} \max_{0 \leq \alpha \leq s_i} \{\deg(f_{i_\alpha})\} &\leq (s_i - 1) (\sum_{\alpha=0}^{s_i} r_1(f_{i_\alpha}) - 1) \\ &\leq (d - 1) \left(\sum_{i=0}^k r_1(f_i) - 1 \right) = (d - 1) (r_1(f_0 \cdots f_k) - 1), \end{aligned}$$

that is, for each $i \in \{0, 1, \dots, k\}$, $\deg(f_i) \leq (d - 1) (r_1(f_0 \cdots f_k) - 1)$. Hence Theorem 1.4 is proved.

Proof of Theorem 1.5. Theorem 1.2 implies

$$\max_{1 \leq j \leq k} \deg(f_j^{l_j}) \leq \max_{0 \leq j \leq k} \deg(f_j^{l_j}) \leq \sum_{j=0}^k r_{k-1}(f_j^{l_j}) - \frac{k(k-1)}{2}, \quad (26)$$

where $l_0 = 1$. Note that

$$r_{k-1}(f_j^{l_j}) \leq (k-1)r_1(f_j) \leq (k-1)\deg(f_j) = \frac{k-1}{l_j} \deg(f_j^{l_j}), \quad j \neq 0. \quad (27)$$

Hence (5) follows from (26) and (27).

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