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TENSOR PRODUCTS AND Λ_0- NUCLEAR SPACES IN P-ADIC ANALYSIS

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Abstract. The Λ_0-nuclearity of the topological tensor product of two Λ_0-nuclear spaces is studied. This problem is related to the question of whether the operator $T_1 \otimes T_2$ is Λ_0-nuclear when T_1 and T_2 are Λ_0-nuclear.

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0. INTRODUCTION Throughout this paper, K will be a complete non-Archimedean valued field whose valuation is nontrivial.

As it shown in [1], if E, F are locally convex spaces over K, then $E \otimes_\pi F$ is nuclear iff E, F are nuclear. In this paper we study the analogous problem for the Λ_0-nuclear spaces which were introduced in [7]. We show that the question is related to each of the following two equivalent conditions :

(1) If $T_1 : E_1 \to F_1$, $T_2 : E_2 \to F_2$ are Λ_0-nuclear operators, then $T_1 \otimes T_2 : E_1 \otimes_\pi E_2 \to F_1 \otimes_\pi F_2$ is Λ_0-nuclear.

(2) If $\xi, \eta \in \Lambda_0 = \Lambda_0(P)$, then there exists a bijection $\sigma = (\sigma_1, \sigma_2) : N \to N \times N$ such that

$$(\xi_{\sigma_1(\eta)}\eta_{\sigma_2(\eta)}) \in \Lambda_0$$

In case the Köthe set P is countable, it is shown that the above conditions are equivalent to :

(3) For each $\alpha \in P$ there exists $\beta \in P$ such that $\sup_n \alpha_{n^2}/\beta_n < \infty$.

1. PRELIMINARIES

By a Köthe set we will mean a collection P of sequences $\alpha = (\alpha_n)$ of non-negative real numbers with the following two properties :

(i) For every $n \in N$ there exists $\alpha \in P$ with $\alpha_n \neq 0$.

(ii) If $\alpha, \alpha' \in P$, then there exists $\beta \in P$ with $\alpha, \alpha' \ll \beta$, where $\alpha \ll \beta$ means that there exists $d > 0$ such that $\alpha_n \leq d\beta_n$ for all n.

For $\alpha \in P$ and $\xi = (\xi_n)$ a sequence in K, we define $p_\alpha(\xi) = \sup_n \alpha_n|\xi_n|$. The non-Archimedean Köthe sequence space $\Lambda(P) = \Lambda$ is the space of all $\xi \in K^N$ such that $p_\alpha(\xi) < \infty$ for all $\alpha \in P$. On $\Lambda(P)$ we consider the locally convex topology generated by the family of non-Archimedean seminorms $\{p_\alpha : \alpha \in P\}$. The subspace $\Lambda_0 = \Lambda_0(P)$ of $\Lambda(P)$ consists of all $\xi \in \Lambda(P)$ such that $\alpha_n|\xi_n| \to 0$ for all $\alpha \in P$.The Köthe set P is called stable if for each $\alpha \in P$ there exists $\beta \in P$ such that $\sup_n \alpha_{2n}/\beta_n < \infty$. By [5, Proposition 2.12], if P is stable and if $\xi, \eta \in \Lambda$ (resp. $\xi, \eta \in \Lambda_0$), then

$$\xi * \eta = (\xi_1, \eta_1, \xi_2, \eta_2, \ldots) \in \Lambda \qquad (resp.\ \xi * \eta \in \Lambda_0).$$

The Köthe set P is called a *power set of infinite type* if

1) For each $\alpha \in P$ we have $0 < \alpha_n \leq \alpha_{n+1}$ for all n.

2) For every $\alpha \in P$ there exists $\beta \in P$ such that $\alpha^2 \ll \beta$.

If $\gamma = (\gamma_n)$ is an increasing sequence and if we take $P = \{(p^{\gamma_n}) : p > 1\}$,then P is a a power set of infinite type. In this case we denote $\Lambda(P)$ by $\Lambda_{\gamma,\infty}$. If $\gamma_n \to \infty$, then for $\Lambda = \Lambda_{\gamma,\infty}$, we have $\Lambda = \Lambda_0$(see [3,, Corollary 3.5]).

Next we will recall the concepts of a Λ_0-compactoid set and a Λ_0 nuclear map, which are given in [5], and the concept of a Λ_0-nuclear space given in [7]. For a bounded subset A, of a locally convex space E over K, and for a non-negative integer n, the nth Kolmogorov diameter $\delta_{n,p}(A)$ of A, with respect to a continuous seminorm p on E ($p \in cs(E)$), is the infimum of all $|\mu|, \mu \in K$, for which there exists a subspace F of E, with $dim F \leq n$, such that $A \subset F + \mu B_p(0,1)$, where

$$B_p(0,1) = \{x \in E : p(x) \leq 1\}.$$

The set A is called Λ_0-compactoid if, for each $p \in cs(E)$, there exists $\xi = \xi_p \in \Lambda_0$ such that $\delta_{n,p}(A) \leq |\xi_{n+1}|$ for all n (or equivalently $\alpha_n \delta_{n-1,p}(A) \to 0$ for each $\alpha \in P$). A continuous linear operator $T : E \to F$ is called :

a) Λ_0-compactoid if there exists a neighborhood V of zero in E such that $T(V)$ is Λ_0-compactoid in F.

b) Λ_0-nuclear if there exist an equicontinuous sequence (f_n) in E', a bounded sequence $(y_n$ in F and $(\lambda_n) \in \Lambda_0$ such that :

$$Tx = \sum_{n=1}^{\infty} \lambda_n f_n(x) y_n \quad (x \in E).$$

For a continuous linear map T, from a normed space E to another one F, and for a non-negative integer n, the nth approximation number $\alpha_n(T)$ of T is defined by

$$\alpha_n(T) = inf\{\|T - A\| : A \in \mathcal{A}_n(E,F)\}$$

where $\mathcal{A}_n(E,F)$ is the collection of all continuous linear operators $A : E \to F$ with $dim A(E) \leq n$.

Throughout the rest of the paper, P will be a Köthe set, which is a power set of infinite type, and $\Lambda_0 = \Lambda_0(P)$.

Let now E be a locally convex space over K. For $p \in cs(E)$, we will denote by E_p the quotient space $E/kerp$ equipped with the norm $\|[x]_p\| = p(x)$. A Hausdorff locally convex space E is called Λ_0-nuclear (see [7]) if for each $p \in cs(E)$ there exists $q \in cs(E)$, $p \leq q$, such that the canonical map $\phi_{pq} : E_q \to E_p$ is Λ_0-nuclear (or equivalently Λ_0-compactoid). If $\phi_q : E \to E_q$ is the quotient map, then $\phi_q(B_q(0,1))$ is the closed unit ball in E_q. It is now clear that E is Λ_0-nuclear iff for each $p \in cs(E)$ the map $\phi_p : E \to E_p$ is Λ_0-nuclear.

Note that if P consists of the single constant sequence $(1,1,\ldots)$, then $\Lambda_0(P) = c_0$ and so in this case the Λ_0-compactoid sets, the Λ_0-compactoid operators and the Λ_0-nuclear operators coincide with the compactoid sets, the compactoid operators and the nuclear operators, respectively. Also, if $T_1 : E \to F, T_2 : F \to G$ are continuous linear maps and if one of the T_1, T_2 is Λ_0-compactoid (resp. Λ_0-nuclear), then T_1, T_2 is Λ_0-compactoid (resp. Λ_0-nuclear) ([5, Proposition 3.21 and Proposition 4.5]). But for normed spaces E, F the class of all Λ_0-nuclear operators from E to F is not necessarily a closed subset of the space of all continuous linear operators from E to F ([6, Corollary 3.7]).

We will denote the completion, of a Hausdorff locally convex space E, by $\widehat{E}$.

We will need a Proposition which is given in [4, Proposition 5.1]. For an index set I, let $c_0(I)$ be the vector space of all $\xi \in K^I$ such that $|\xi_i| \to 0$, i.e. for each $\epsilon > 0$ the set $\{i \in I : |\xi_i| > \epsilon\}$ is finite. On $c_0(I)$ we consider the norm $\|\xi\| = \sup_i |\xi_i|$.

Proposition 0.1 : *Let $\zeta = (\zeta_i)$ be a fixed element of $c_0(I)$ and consider the map*

$$T : c_0(I) \to c_0(I), \ (T\xi)_i = (\xi_i \zeta_i).$$

Then, for each non-negative integer n we have

$$\alpha_n(T) = \sup_{J \in \mathcal{F}_{n+1}} \inf_{i \in J} |\zeta_i|$$

where $\mathcal{F}_{n+1}$ is the collection of all subsets of I containing $n+1$ elements.

2. ON THE Λ_0-NUCLEAR MAPS

For a fixed $\xi \in c_0$, the map $T_\xi : c_0 \to c_0$ is defined by $(T_\xi x)_i = \xi_i x_i$ for each $x \in c_0$. As it easy to see, if $\xi \in \Lambda_0$, then T_ξ is Λ_0-nuclear.

Proposition 2.1 : *Let E, F be locally convex spaces over K, where F is complete, and let $T : E \to F$ be a Λ_0-nuclear map. Then, there exist $\xi \in \Lambda_0$ and continuous linear maps $T_1 : E \to c_0, T_2 : c_0 \to F$ such that $T = T_2 T_\xi T_1$.*

Proof : Let $(\lambda_n) \in \Lambda_0$, (f_n) an equicontinuous sequence in E' and (y_n) a bounded sequence in F be such that $Tx = \sum_n \lambda_n f_n(x) y_n$ for all $x \in E$. Let $|\lambda| > 1$ and choose $\mu_n \in K$ such that $|\mu_n| \leq \sqrt{|\lambda_n|} \leq |\lambda \mu_n|$. As it is shown in the proof of Theorem 4.6 in [5], $(\mu_n) \in \Lambda_0$. Let $\xi = (\xi_n)$ where $\xi_n = 0$ if $\mu_n = 0$ and $\xi_n = \lambda_n \mu_n^{-1}$ if $\mu_n \neq 0$. Then $(\xi_n) \in \Lambda_0$. Define

$$T_1 : E \to c_0, \ T_1 x = (\mu_n f_n(x)).$$

Let $D = (T_\xi T_1)(E)$. If $\bar{D}$ is the closure of D in c_0, then there exists a projection Q of c_0 onto $\bar{D}$ with $\|Q\| \leq |\lambda|$ (see [10, Theorem 3.16]). Let $S : D \to F, S(T_\xi T_1 x) = Tx$. Then S is well defined and continuous. Let $\bar{S} : \bar{D} \to F$ be the continuous extension of S and define $T_2 : c_0 \to F, T_2 = \bar{S}Q$. Now $T = T_2 T_\xi T_1$

Lemma 2.2 : *Let $\xi = (\xi_n) \in K^N$ be such that $|\xi_n \geq |\xi_{n+1}|$ for all n. If there exists a permutation σ of N such that $(\xi_{\sigma(n)}) \in \Lambda_0$, then $\xi \in \Lambda_0$*

Proof. Let $\zeta = (\xi_{\sigma(n)})$ and let $T = T_\zeta : c_0 \to c_0$. Since $\zeta \in \Lambda_0$, T is Λ_0-nuclear. In view of [5, Theorem 4.1], T is of type Λ_0 and so there exists $(\mu_n) \in \Lambda_0$ such that $\alpha_n(T) \leq |\mu_{n+1}|$ for all n. Using Proposition 0.1, we get that $\alpha_n(T) = |\xi_{n+1}|$, which clearly implies that $\xi \in \Lambda_0$.

Definition 2.3 : *Let Let $\xi = (\xi_n) \in K^N$. A sequence $\zeta = (\zeta_n)$ is called a decreasing rearrangement of ξ if :*

a)$|\zeta_n| \geq |\zeta_{n+1}|$, for all n.

b) There exists a permutation σ on N such that $\zeta_n = \xi_{\sigma(n)}$ for all n.

It is easy to see that if (ζ_n) and (μ_n) are decreasing rearrangements of ξ, then $|\zeta_n| = |\mu_n|$ for all n,

Proposition 2.4 : *Let Let $\xi = (\xi_n) \in c_0$ with $\xi_n \neq 0$ for all n. Then :*

a) There exists a decreasing rearrangement of ξ.

b) If $\xi \in \Lambda_0$ and if $(\xi_{\sigma(n)})$ is any decreasing rearrangement of ξ, then $(\xi_{\sigma(n)}) \in \Lambda_0$.

Proof : a) Let n_1 be the first of all indices k with $|\xi_k| = \sup_m |\xi_m| = \max_m |\xi_m|$. Having chosen $n_1, n_2, \ldots, n_m$, let n_{m+1} be the first index $k \neq n_1, n_2, \ldots, n_m$ with $|\xi_k| = max\{|\xi_n| : n \neq n_1, n_2, \ldots, n_m\}$. Let $\sigma : N \to N, \sigma(m) = n_m$. We claim that $(\xi_{\sigma(n)})$ is a decreasing rearrangement of ξ. Since $|\xi_{n_m}| \geq |\xi_{n_{m+1}}|$ for all m, it only remains to show that $\sigma(N) = N$. So, let $m \in N$ and suppose $m \notin \sigma(N)$. For each $k \in N$, since $m \neq n_1, n_2, \ldots, n_{k-1}$, we have $|\xi_m| \leq |\xi_{n_k}|$. This contradicts the fact that the set $N_1 = \{k : |\xi_k| \geq |\xi_m|\}$ is finite.

b) It follows from Lemma 2.2.

Let ϕ be the subspace of Λ_0 consisting of all sequences in K with only a finite number of non-zero terms. Suppose that $\Lambda_0 \neq \phi$ (this for instance happens when P is countable by [6, Remark 4,4]). If $\xi \in \Lambda_0 \backslash \phi$ and if $\mu_n \in K, |\mu_n| = \sup_{k \geq n} |\xi_k|$, then $(\mu_n) \in \Lambda_0$ and $\mu_n \neq 0$ for all n.

Proposition 2.5 : *Let E, F be locally convex spaces, where F is metrizable and let G be a dense subspace of F. Let $T \in L(E,F)$ be Λ_0-nuclear and suppose that P is stable and that $\Lambda_0 \neq \phi$. Then, there exist $(\xi_n) \in \Lambda_0$, an equicontinuous sequence (g_n) in E' and a bounded sequence (z_n) in G such that*

$$Tx = \sum_n \xi_n g_n(x) z_n \;\; (x \in E)$$

Proof : Let (p_m) be an increasing sequence of continuous seminorms on F generating its topology. Since G is dence in $\widehat{F}$, we may assume that F is complete. Let $(\lambda_n) \in \Lambda_0$, $0 < |\lambda_{n+1}| \leq |\lambda_n|$. Since T is Λ_0-nuclear, there exist $(\mu_n) \in \Lambda_0$, (h_n) an an equicontinuous sequence in E' and a bounded sequence (y_n) in F such that $Tx = \sum_n \mu_n h_n(x) y_n$. We may assume that $|\mu_n| \leq 1$ for all n. For each positive integer n, there are unique positive integers k, m such that $n = (2m-1)2^{k-1}$. Set $\xi_m^{(k)} = \lambda_{(2m-1)2^{k-1}}$. Choose $z_m^{(k)} \in G$ such that

$$max\{p_m(z_m^{(k)} - y_k), p_k(z_m^{(k)} - y_k)\} \leq |\xi_{m+1}^{(k)}|.$$

Set $w_1^{(k)} = z_1^{(k)}$ and $w_m^{(k)} = z_m^{(k)} - z_{m-1}^{(k)}$ if $m \geq 2$. For all k, we have $y_k = \lim_{m \to \infty} z_m^{(k)}$. Indeed, let $n \in N$. If $m \geq n$, then

$$p_n(z_m^{(k)} - y_k) \leq p_m(z_m^{(k)} - y_k) \leq |\xi_{m+1}^{(k)}| \to 0 \;\; as \;\; m \to \infty.$$

Since $\sum_{i=1}^m w_i^{(k)} = z_m^{(k)}$, we have that $y_k = \sum_{m=1}^\infty w_m^{(k)}$. Thus, for all $x \in E$, we have

$$Tx = \sum_k \mu_k h_k(x) y_k = \sum_k \sum_m \mu_k h_k(x) w_m^{(k)}.$$

Let $v_1^{(k)} = w_1^{(k)}$, $\eta_1^{(k)} = 1$.For $m \geq 2$, let $v_m^{(k)} = w_m^{(k)} / \xi_m^{(k)}$, $\eta_m^{(k)} = \xi_m^{(k)}$. The set $\{v_m^{(k)} : m \geq 2, k \in N\}$ is bounded in G. In fact, let $n \in N$. If $k > n$, then

$$\begin{aligned} p_n(w_m^{(k)}) &= max\{p_k(z_m^{(k)} - y_k), p_k(z_{m-1}^{(k)} - y_k)\} \\ &\leq max\{|\xi_{m+1}^{(k)}|, |\xi_m^{(k)}| = |\xi_m^{(k)}|. \end{aligned}$$

Similarly, for $m > n$, we have

$$p_n(w_m^{(k)}) \leq max\{p_m(z_m^{(k)} - y_k), p_{m-1}(z_{m-1}^{(k)} - y_k)\} \leq |\xi_m^{(k)}|.$$

Also, the set $\{v_1^{(k)} : k \in N\} = \{z_1^{(k)} : k \in N\}$ is bounded since, for $n \in N$ and $k > n$ we have

$$p_n(z_1^{(k)}) \leq max\{p_k(z_1^{(k)} - y_k), p_n(y_k)\} \leq max\{|\xi_2^{(k)}|, p_n(y_k)\}$$

and so $\sup_k p_n(z_1^{(k)}) < \infty$ since (y_k) and (λ_m) are bounded. Let

$$\{n_1 < n_2 < \ldots\} = \{(2m-1)2^{k-1} : k \in N, m \geq 2\}.$$

For $i \in N$, set $\xi_i = \mu_k \lambda_{(2m-1)2^{k-1}}, f_i = h_k$ and $z_i = v_m^{(k)}$ if $n_i = (2m-1)2^{k-1}$. Since every subsequence of (λ_n) is in Λ_0 and since $|\mu_k| \leq 1$ for all k, it is clear that $\xi = (\xi_i) \in \Lambda_0$. Let $\zeta_k = \mu_k, w_k = z_1^{(k)}$. If $\zeta = (\zeta_k)$ then $\xi * \zeta \in \Lambda_0$ since P is stable. Moreover

$$Tx = \xi_1 f_1(x) z_1 + \zeta_1 h_1(x) w_1 + \xi_2 f_2(x) z_2 + \zeta_2 h_2(x) w_2 + \ldots.$$

This completes the proof.

Proposition 2.6 : *Let F be a dense subspace of a Hausdorff locally convex space over K. Then, E is Λ_0-nuclear iff F is Λ_0-nuclear.*

Proof : In view of [7, Proposition 3.4], a locally convex space M is Λ_0-nuclear iff every continuous linear map from M to any Banach space G is Λ_0-nuclear. Now the result follows easily from this and the fact that every continuous linear map, from F to any Banach space, has a continuous extension to all of E.

3. TENSOR PRODUCTS AND Λ_0-NUCLEAR SPACES

Proposition 3.1 : *Let P be countable. Then, the following are equivalent :*

(1) P is stable.

(2) For all $\xi, \eta \in \Lambda_0$ we have $\xi * \eta \in \Lambda_0$.

(3) For every $\xi \in \Lambda_0$ we have $\xi * \xi \in \Lambda_0$.

(4) If $\xi, \eta \in \Lambda_0$, then some rearrangement of the sequence $\xi * \eta$ is in Λ_0.

(5) If $\xi \in \Lambda_0$, then some rearrangement of $\xi * \xi$ is in Λ_0.

Proof : (1) implies (2) by [5, Proposition 2.12].

(3) $\Rightarrow$ (4). Let $\zeta_n \in K, |\zeta_n| = max\{|\xi_n|, |\eta_n|\}$. Then $\zeta = (\zeta_n) \in \Lambda_0$. Since $\zeta * \zeta \in \Lambda_0$, it is clear that $\xi * \eta \in \Lambda_0$.

(5) $\Rightarrow$ (1). Let $|\lambda| > 1$. Without loss of generality, we may assume that $P = \{\alpha^n : n \in N\}, |\lambda|\alpha^n \leq \alpha^{n+1}$.

Suppose that P is not stable and let $\alpha \in P$ be such that $\sup_n \alpha_{2n}/\beta_n = \infty$ for every $\beta \in P$. Choose indices $n_1 < n_2 < \ldots$ such that $\alpha_{2n_k}/\alpha_{n_k}^{(k)} > k$ for all k. There are $\lambda_k \in K$ with

$$|\lambda^{-1}\lambda_k \leq (k\alpha_{n_k}^{(k)})^{-1} \leq |\lambda_k|.$$

Let $n_0 = 0$ and for $n_{k-1} < n \leq n_k$ set $\xi_n = \lambda_k$. Now, for every $k \in K$ we have $|\lambda_{k+1}| \leq |\lambda_k|$. Moreover $\xi = (\xi_n) \in \Lambda_0$. In fact, if $k_0 \in N$, then for $k \geq k_0$ we have

$$\alpha_{n_k}^{(k_0)} |\xi_{n_k}| \leq \alpha_{n_k}^{(k)} |\xi_{n_k}| \leq |\lambda|/k \to 0.$$

By our assumption (5), there exists a rearrangement of the sequence $(\gamma_n) = \xi * \xi$ which belongs to Λ_0. This, and the fact that $|\gamma_n| \geq |\gamma_{n+1}|$ for all n, imply that $(\gamma_n) \in \Lambda_0$ (by Lemma 2.2). But $\alpha_{2n_k} |\xi_{n_k}| \geq k\alpha_{n_k}^{(k)} (k\alpha_{n_k}^{(k)})^{-1} = 1$, a contradiction.

Proposition 3.2 : *Let P be countable and suppose that for each $\xi \in \Lambda_0$ there exists a bijection $\sigma = (\sigma_1, \sigma_2) : N \to N \times N$ such that $(\xi_{\sigma_1(n)} \xi_{\sigma_2(n)}) \in \Lambda_0$. Then, P is stable.*

Proof : Let $|\lambda| > 1$. Without loss of generality, we may assume that $P = \{\alpha^{(n)} : n \in N\}, |\lambda| \alpha^{(n)} \leq \alpha^{(n+1)}$ for all n. Suppose that P is not stable and let $\alpha \in P$ be such that $\sup_n \alpha_{2n}/\beta_n = \infty$ for all $\beta \in P$. As in the proof of the implication (5) $\Rightarrow$ (1) in the preceding proposition, let $n_0 = 0 < n_1 < \ldots$ be such that $\alpha_{2n_k}/\alpha_{n_k}^{(k)} > k$ and let $|\lambda^{-1}\lambda_k \leq (k\alpha_{n_k}^{(k)})^{-1} \leq |\lambda_k|$. If $n_{k-1} < n \leq n_k$, set $\xi_n = \lambda_k$. Then $(\xi_n) \in \Lambda_0$. By our hypothesis there is some rearrangement of the sequence

$$\zeta = (\xi_1\xi_1, \xi_1\xi_2, \xi_2\xi_1, \xi_1\xi_3, \xi_2\xi_2, \xi_3\xi_1, \ldots)$$

which belongs to Λ_0. In view of Lemma 2.2, if (γ_n) is a decreasing rearrangement of ζ, then $(\gamma_n) \in \Lambda_0$. Consider the sequence

$$\eta = (\xi_1\xi_1, \xi_2\xi_1, \xi_2\xi_2, \xi_1\xi_2, \xi_3\xi_1, \xi_1\xi_3, \ldots, \xi_n\xi_1, \xi_1\xi_n, \ldots)$$

and let (δ_n) be a decreasing rearrangement of η. Then $|\delta_k| \leq |\gamma_k|$ for all k. In fact, suppose that $|\delta_k| > |\gamma_k|$ for some k. Then $|\delta_1| \geq |\delta_2| \geq \ldots \geq |\delta_k| > |\gamma_k|$. Since $|\gamma_m| \leq |\gamma_k| < |\delta_k|$ for all $m \geq k$, we must have that

$$\{\delta_1, \ldots, \delta_k\} \subset \{\gamma_1, \ldots, \gamma_{k-1}\}$$

which clearly is a contradiction. Thus, $|\delta_k| \leq |\gamma_k|$ for all k, and so $(\delta_n) \in \Lambda_0$. let $\mu \in K, |\mu| = min\{|\xi_1|, |\xi_2|\}$, and consider the sequence

$$(\lambda_n) = (\xi_1, \xi_1, \xi_2, \xi_2, \xi_3, \xi_3, \ldots) = \xi * \xi.$$

Since $|\eta_n| \geq |\mu\lambda_n|$ for all n, there exists some rearrangement of (λ_n) which belongs to Λ_0 and so $(\lambda_n) \in \Lambda_0$ since $|\lambda_n| \geq |\lambda_{n+1}|$ for all n. Since $\alpha_{2n_k} |\xi_{n_k} \geq 1$, we got a contradiction. This clearly completes the proof.

Proposition 3.3 : *Consider the following conditions :*

(1) For each $\alpha \in P$ there exists $\beta \in P$ such that $\sup_n \alpha_{n^2}/\beta_n < \infty$.

(2) If $\xi, \eta \in \Lambda_0$, then there exists a bijection $\sigma = (\sigma_1, \sigma_2) : N \to N \times N$ such that $(\xi_{\sigma_1(n)}\xi_{\sigma_2(n)}) \in \Lambda_0$.

(3) If $\xi \in \Lambda_0$, then there exists a bijection $\sigma = (\sigma_1, \sigma_2) : N \to N \times N$ such that $(\xi_{\sigma_1(n)}\xi_{\sigma_2(n)}) \in \Lambda_0$.

Then (1) $\Rightarrow$ (2) $\Rightarrow$ (3). If P is countable, then (1), (2), (3) are equivalent.

Proof: (1) $\Rightarrow$ (2). Let $\sigma = (\sigma_1, \sigma_2) : N \to N \times N$ be defined as follows : Let $\sigma(1) = (1,1)$. For $j = [1 + 2 + \ldots + (n-1)] + k = \frac{n(n-1)}{2} + k, 1 \le k \le n$, let $\sigma(j) = (k, n+1-k)$. Then $(\lambda_n) = (\xi_{\sigma_1(n)}\xi_{\sigma_2(n)}) \in \Lambda_0$. In fact, let $\alpha \in P$. Our assumption on P implies that P is stable. Thus, there exists $\beta \in P$ such that $\sup_n \alpha_{2n^2}/\beta_n = d < \infty$. Let $d_1 > 0$ be such that $|\xi_k|, |\eta_k| \le d_1$ for all k. Let $\epsilon > 0$ be given and choose n_0 such that $\beta_k|\xi_k|, \beta_k|\eta_k| < \frac{\epsilon}{dd_1}$ if $k \ge k_0$. Let now $j > \frac{m(m-1)}{2}$, where $m \ge 2k_0$, and let $j = \frac{n(n-1)}{2} + k, 1 \le k \le n$. Clearly $n \ge m$. We have that either $k \ge \frac{n+1}{2}$ or $n+1-k \ge \frac{n+1}{2}$. If, say, $k \ge \frac{n+1}{2}$, then $j \le \frac{n(n+1)}{2} \le 2k^2$ and $\alpha_j|\xi_k\eta_{n+1-k}| \le d_1\alpha_{2k^2}|\xi_k| \le d_1 d\beta_k|\xi_k| < \epsilon$ since $k \ge \frac{n+1}{2} \ge \frac{m+1}{2} > k_0$. The same happens when $n+1-k \ge \frac{n+1}{2}$. Thus, for $j > \frac{m(m-1)}{2}$, we have $|\alpha_j\lambda_j| < \epsilon$, which proves that $(\lambda_n) \in \Lambda_0$.

Assume next that P is countable and that (3) holds. Let $|\lambda| > 1$. Without loss of generality we may assume that From : Athanasios Katsaras ¡akatsar@cc.uoi.gr¿ Organization : University of Ioannina Computer Center Dourouti, Ioannina, Greece 451 10 tel : +30-651-45298, fax : +30-651-45298 Date : Wed, 12 Oct 94 12 :32 :30 +0200 To : escassut@ucfma, katsara@cc.uoi.gr

$$P = \{\alpha^{(n)} : n = 0, 1, \ldots\}, [\alpha^{(n-1)}]^2 \le \alpha^{(n)}, |\lambda|\alpha^{(n)} \le \alpha^{(n+1)}$$

$\alpha_1^{(0)} \ge 1$. Suppose that (1) does not hold and let $\alpha \in P$ be such that $\sup_n \alpha_{n^2}/\beta_n = \infty$ for all $\beta \in P$. Let (n_k) be a sequence of natural numbers, with $n_k > 2n_{k-1}$, such that $\alpha_{n_k^2}/\alpha_{n_k}^{(k)} > k^2$ for $k = 1, 2, \ldots$. Choose $\lambda_k \in K$ with

$$|\lambda^{-1}\lambda_k| \le (k\alpha_{n_k}^{(k-1)})^{-1} \le |\lambda_k|.$$

Let $n_0 = 0$ and, for $n_{k-1} < n \le n_k$, let $\xi_n = \lambda_k$. If $k \ge k_0 + 1$, then

$$\alpha_{n_k}^{(k_0)}|\xi_{n_k}| \le \alpha_{n_k}^{(k-1)}|\lambda|(k\alpha_{n_k}^{(k-1)})^{-1} = \frac{|\lambda|}{k} \to 0 \ \ as \ \ k \to \infty.$$

This proves that $(\xi_n) \in \Lambda_0$. Also,

$$|\xi_{n_{k+1}}| \le |\lambda|((k+1)\alpha_{n_{k+1}}^{(k)})^{-1} \le (k\alpha_{n_k}^{(k-1)})^{-1} \le |\xi_{n_k}|.$$

Let $I_k = \{n : n_{k-1} < n \le n_k\}$. If $i, j \in I_k$, then $|\xi_i\xi_j| = |\xi_{n_k}^2|$. Let

$$\zeta = (\xi_1\xi_1, \xi_1\xi_2, \xi_2\xi_1, \xi_1\xi_3, \xi_2\xi_2, \xi_3\xi_1, \ldots)$$

and let $\eta = (\eta_1, \eta_2, \ldots)$ be the sequence which we get by writing first those $\xi_i\xi_j$ with $i, j \in I_1$, then those $i, j \in I_2$ e.t.c. Clearly $|\eta_1| \geq |\eta_2| \geq \ldots$. By our hypothesis (3), there exists a rearrangement, of the terms of the sequence ζ, which belongs to Λ_0. This implies that any decreasing rearrangement (μ_n) of ζ also belongs to Λ_0. Now, for every k, we have $|\mu_k| \geq |\eta_k|$. In fact, if $|\mu_k| < |\eta_k|$, for some k, then

$$\{\eta_1, \eta_2, \ldots, \eta_k\} \subset \{\mu_1, \mu_2, \ldots, \mu_{k-1}\},$$

a contradiction. Hence $|\mu_m| \geq |\eta_m|$, for all m and so $(\eta_k) \in \Lambda_0$. The number of the terms $\xi_i\xi_j$, with $i, j \in I_k$, is $(\eta_k - \eta_{k-1})^2$. Let $m_1 = n_1^2, m_k = m_{k-1} + (\eta_k - \eta_{k-1})^2$ for $k \geq 2$. Since $n_k > 2n_{k-1}$, we have $n_k - n_{k-1} > \frac{n_k}{2}$ and so $m_k > \frac{n_k^2}{4}$. In view of Proposition 3.2, there exists $\beta \in P$ and $\mu \in K$ with $\alpha_{4n}/\beta_n \leq |\mu|$ for all n. Now

$$\begin{aligned} \beta_{m_k}|\eta_{m_k}| &= \beta_{m_k}|\xi_{n_k}|^2 \geq |\mu|^{-1}\alpha_{4m_k}|\xi_{n_k}|^2 \\ &\geq |\mu|^{-1}\alpha_{n_k^2}|\xi_{n_k}|^2 \geq |\mu|^{-1}k^2\alpha_{n_k}^{(k)}|\xi_{n_k}|^2 \\ &\geq |\mu|^{-1}k^2(\alpha_{n_k}^{(k-1)})^2|\xi_{n_k}|^2 \geq |\mu|^{-1}, \end{aligned}$$

which contradicts the fact that $(\eta_m) \in \Lambda_0$. This clearly completes the proof.

Proposition 3.4 : *Let $\psi : c_0 \times c_0 \to c_0(N \times N)$ be defined by $\psi(x, y) = (x_i y_j)_{i,j}$ for $x = (x_i), y = (y_i)$. Then*

(1) ψ is a continuous bilinear map and $\|\psi(x, y)\| = \|x\|\|y\|$.

(2) If $\widetilde{\psi} : c_0 \otimes_\pi c_0 \to c_0(N \times N)$ is the corresponding linear map, then $\widetilde{\psi}$ is an isometry and $D = \widetilde{\psi}(c_0 \otimes_\pi c_0)$ is dense in $c_0(N \times N)$.

(3) The continuous extension $\omega : c_0\widehat{\otimes}_\pi c_0 \to c_0(N \times N)$ of $\widetilde{\psi}$ is an onto isometry.

Proof : (1) It is trivial.

(2) Let $u \in c_0 \otimes_\pi c_0$ and let p the norm on c_0 and set $\|.\| = p \otimes_\pi p$. If $u = \sum_{k=1}^m x^k \otimes_\pi y^k$, then

$$\|\widetilde{\psi}(u)\| \leq \max_k \|\widetilde{\psi}(x^k \otimes y^k)\| = \max_k \|\psi(x^{(k)}, y^{(k)})\| = \max_k p(x^{(k)})p(y^{(k)})$$

and so $\|\widetilde{\psi}(u)\| \leq \|u\|$. On the other hand, given $0 < t < 1$, there are t-orthogonal elements $y^{(1)}, \ldots, y^{(n)}$ of c_0 and $x^{(1)}, \ldots, x^{(n)} \in c_0$ such that $u = \sum_{k=1}^n x^k \otimes y^k$. Thus

$$\begin{aligned} \|\widetilde{\psi}(u)\| &= \sup_{i,j} \|\sum_{k=1}^n x_i^k y_j^k\| \\ &= \sup_i[\sup_j |x_i^1 y_j^1 + x_i^2 y_j^2 + \ldots x_i^n y_j^n|] \\ &= \sup_i p(x_i^1 y^{(1)} + x_i^2 y^{(2)} + \ldots x_i^n y^{(n)}) \\ &\geq t \sup_i \max_{1 \leq k \leq n} |x_i^{(k)}| p(y^{(k)}) = t \max_{1 \leq k \leq n} p(x^{(k)})p(y^{(k)}) \geq t\|u\|. \end{aligned}$$

Since $0 < t < 1$ was arbitrary, we have that $\|\widehat{\psi}(u)\| \geq \|u\|$ and so $\|\widehat{\psi}(u)\| = \|u\|$. To see that D is dense in $c_0(N \times N)$, let $w = (\xi_{ij})_{i,j} \in c_0(N \times N)$ and let $\epsilon > 0$. Choose m such that $|\xi_{ij}| < \epsilon$ if $i > m$ or $j > m$. Let $w_0 = (\mu_{ij})$ with $\mu_{ij} = \xi_{ij}$ if $i, j \leq m$ and $\mu_{ij} = 0$ if $i > m$ or $j > m$. Then $w_0 \in D$ and $\|w - w_0\| \leq \epsilon$.

(3) If $u \in c_0 \widehat{\otimes}_\pi c_0$, then there exists a sequence $(u^{(n)})$ in $c_0 \otimes_\pi c_0$ converging to u. Now

$$\|\omega(u)\| = \lim_n \|\widetilde{\psi}(u^{(n)})\| = \lim_n \|u^{(n)}\| = \|u\|$$

and so u is an isometry. This and the fact that $\omega(c_0 \widehat{\otimes}_\pi c_0)$ is dense in $c_0(N \times N)$ imply that ω is onto.

Proposition 3.5 *Let E, F be locally convex spaces over $K, E, F \neq \{0\}$. If $E \otimes F$ is Λ_0-nuclear, then E and F are Λ_0-nuclear.*

Proof. Since $E \otimes_\pi F$ is Λ_0-nuclear, it is by definition Hausdorff which implies that both E and F are Hausdorff. Let now $p \in cs(E)$ and choose $y_0 \in F$ and $q \in cs(F)$ such that $q(y_0) \neq 0$. Since $E \otimes_\pi F$ is Λ_0-nuclear, there exist (by [7, Proposition 3.4]) $(\lambda_n) \in \Lambda_0$ and an equicontinuous sequence h_n in $(E \otimes_\pi F)'$ such that

$$p \otimes q(u) \leq \sup_n |\lambda_n h_n(u)| \ \ (u \in E \otimes_\pi F).$$

Let $f_n : E \to K$, $f_n(x) = h_n(x \otimes y_0)$. Then (f_n) is an equicontinuous sequence in E'. Let $\mu \in K$ with $q(y_0) \geq |\mu|^{-1}$. Then

$$p(x) \leq |\mu| \sup_n |\lambda_n f_n(x)| \ \ (x \in E)$$

Thus E is Λ_0-nuclear (by [7, Proposition 3.4]). The proof of the Λ_0-nuclearity of F is analogous.

If E_1, E_2, F_1, F_2 are locally convex spaces over K and if $T_i : E_i \to F_i, i = 1, 2$, are linear maps, then $T_1 \otimes T_2 : E_1 \otimes E_2 \to F_1 \otimes F_2$ will be defined by

$$T_1 \otimes T_2(x \otimes y) = T_1(x) \otimes T_2(y).$$

We will denote by $N_{\Lambda_0}(E, F)$ the collection of all Λ_0-nuclear operators from E to F. Recall also that for $\xi \in c_0, T_\xi : c_0 \to c_0$ is defined by $(T_\xi x)_k = \xi_k x_k$.

Theorem 3.6 : *Consider the following properties :*

(1) If E_1, E_2, F_1, F_2 are locally convex spaces over K, where F_1, F_2 are Hausdorff, and if $T_i \in N_{\Lambda_0}(E_i, F_i), i = 1, 2$, then $T_1 \otimes T_2 \in N_{\Lambda_0}(E_1 \otimes_\pi E_2, F_1 \otimes_\pi F_2)$.

(2) If $\xi, \eta \in \Lambda_0$, then $T_\xi \otimes T_\eta \in N_{\Lambda_0}(c_0 \otimes_\pi c_0, c_0 \otimes_\pi c_0)$.

(3) If $\xi \in \Lambda_0$, then $T_\xi \otimes T_\xi \in N_{\Lambda_0}(c_0 \otimes_\pi c_0, c_0 \otimes_\pi c_0)$.

(4) If $\xi, \eta \in \Lambda_0$, then there exists a bijection $\sigma = (\sigma_1, \sigma_2) : N \to N \times N$ such that $(\xi_{\sigma_1(n)}\eta_{\sigma_2(n)} \in \Lambda_0$.

(5) If $\xi \in \Lambda_0$, then there exists a bijection $\sigma = (\sigma_1, \sigma_2) : N \to N \times N$ such that $(\xi_{\sigma_1(n)}\xi_{\sigma_2(n)} \in \Lambda_0$.

(6) If E, F are Λ_0-nuclear spaces, then $E \otimes_\pi F$ is Λ_0-nuclear.

Then, (1)-(5) are equivalent and they imply (6).

Proof : Since, for $\xi \in \Lambda_0$, T_ξ is Λ_0-nuclear , it is clear that (1) implies (2).

(3) $\Rightarrow$ (4) Let $\mu_n \in K$ with $|\mu_n| = max\{|\xi_n|, |\eta_n|\}$. Then $\zeta = (\mu_n) \in \Lambda_0$. If there exists a bijection $\sigma = (\sigma_1, \sigma_2) : N \to N \times N$ such that $(\xi_{\sigma_1(n)}\eta_{\sigma_2(n)} \in \Lambda_0$, then $(\xi_{\sigma_1(n)}\eta_{\sigma_2(n)} \in \Lambda_0$.

Thus, we may assume that $\xi = \eta$. If now ξ has only a finite number of nonzero terms, then it is clear that $(\xi_{\sigma_1(n)}\eta_{\sigma_2(n)} \in \Lambda_0$ for any bijection $\sigma = (\sigma_1, \sigma_2) : N \to N \times N$. So, we may assume that the set $\{n : \xi_n \neq 0\}$ is infinite. If $\mu_n \in K$, $|\mu_n| = \sup_{k \geq n} |\xi_k|$, then $(\mu_n) \in \Lambda_0$. It is clear that if we prove the result for (μ_n), then it would also hold for ξ. Thus, we may assume that $0 < |\xi_{n+1}| \leq |\xi_n|$ for all n. Let $T = T_\xi$. By our hypothesis $T \otimes T \in N_{\Lambda_0}(c_0 \otimes_\pi c_0, c_0 \otimes_\pi c_0)$. Let $\omega : c_0 \widehat{\otimes_\pi} c_0 \to c_0(N \times N)$ be the onto isometry in Proposition 3.4. Since $T \otimes T$ is Λ_0-nuclear, the same is true with the continuous extension $T \widehat{\otimes} T : c_0 \widehat{\otimes_\pi} c_0 \to c_0 \widehat{\otimes_\pi} c_0$. In view of [5, Proposition 4.5], the map

$$S = \omega(T \widehat{\otimes} T)\omega^{-1} : c_0(N \times N) \to c_0(N \times N)$$

is Λ_0-nuclear. It is easy to see that for every $w = (w_{i,j})$ in $c_0(N \times N)$ we have $S(w) = (\xi_i \xi_j w_{ij})$. Let

$$\zeta = (\xi_1\xi_1, \xi_1\xi_2, \xi_2\xi_1, \xi_1\xi_3, \xi_2\xi_2, \xi_3\xi_1, \ldots)$$

and let (μ_n) be a decreasing rearrangement of ζ.

It is clear that there exists some bijection $\sigma = (\sigma_1, \sigma_2) : N \to N \times N$ such that $\mu_n = \xi_{\sigma_1(n)}\xi_{\sigma_2(n)}$ for all n. So it suffices to show that $(\mu_n) \in \Lambda_0$. If $\mathcal{F}_{n+1}$ is the family of all subsets J of $N \times N$ containing $n + 1$ elements, then

$$\alpha_n(S) = \sup_{J \in \mathcal{F}_{n+1}} inf_{(i,j) \in J} |\xi_i \xi_j|$$

by Proposition 0.1. Since $|\mu_k| \geq |\mu_{k+1}|$ for all k, it is clear that $\alpha_n(S) = |\mu_{n+1}|$. Thus $(\mu_n) \in \Lambda_0$ since S is Λ_0-nuclear and hence of type Λ_0 (see[5, Theorem 4.2]). This completes the proof of the implication (1) $\Rightarrow$ (4).

(5) $\Rightarrow$ (1). Let $E_1, E_2, F_1, F_2, T_1, T_2$ be as in (1). Since $T_1 : E_1 \to \widehat{F}_1$ and $T_2 : E_2 \to \widehat{F}_2$ are Λ_0-nuclear, there are (by Proposition 2.1) $\gamma = (\gamma_n), \delta = (\delta_n) \in \Lambda_0$ and continuous linear maps $S_1 : E_1 \to c_0, S_2 : c_o \to \widehat{F}_1, H_1 : E_2 \to c_0, H_2 : c_o \to \widehat{F}_2$ such that

$$T_1 = S_2 T_\gamma S_1 \quad and \quad T_2 = H_2 T_\delta H_1.$$

Now

$$T_1 \otimes T_2 = (S_2 \otimes H_2)(T_\gamma \otimes T_\delta)(S_1 \otimes H_1).$$

In order to show that $T_1 \otimes T_2$ is Λ_0-nuclear, it suffices (by [5, Proposition 4.5]) to show that

$$S = T_\gamma \otimes T_\delta : c_0 \otimes_\pi c_0 \to c_0 \otimes_\pi c_0$$

is Λ_0-nuclear. For this, it is enough to show that the continuous extension

$$\widehat{S} : c_0 \widehat{\otimes}_\pi c_0 \to c_0 \widehat{\otimes}_\pi c_0$$

is Λ_0-nuclear. Let $\omega : c_0 \widehat{\otimes}_\pi c_0 \to c_0(N \times N)$ be the onto isometry defined in proposition 3.4 and let

$$H = \omega \widehat{S} \omega^{-1} : c_0(N \times N) \to c_0(N \times N)$$

Since $\widehat{S} = \omega^{-1} H \omega$, it suffices to show that H is Λ_0-nuclear. It is easy to see that (5) implies (4). Thus, our hypothesis (5) implies that there exists a bijection $\sigma = (\sigma_1, \sigma_2) : N \to N \times N$ such that $(\gamma_{\sigma_1(n)} \delta_{\sigma_2(n)}) \in \Lambda_0$. For each $n \in N$, let $f_n \in c_0(N \times N)'$ be defined by $f_n(w) = w_{\sigma_1(n)\sigma_2(n)}$ and let $z^{(n)} \in c_0(N \times N)$, where $z_{ij}^{(n)} = 1$ if $(i,j) = \sigma(n)$ and $z_{ij}^{(n)} = 0$ if $(i,j) \neq \sigma(n)$. Now, $(z^{(n)})$ is a bounded sequence in $c_0(N \times N)$, (f_n) an equicontinuous sequence in $c_0(N \times N)'$ and

$$H(w) = \sum_{n=1}^{\infty} \xi_n f_n(w) z^{(n)}, \quad \xi_n = \gamma_{\sigma_1(n)} \delta_{\sigma_2(n)}.$$

Thus H is Λ_0-nuclear, which proves the implication (5) $\Rightarrow$ (1).

(1) $\Rightarrow$ (6). Let p, q be continuous seminorms on E and F, respectively, and $r = p \otimes q$. Consider the canonical linear isometry

$$h = E_p \otimes_\pi E_q \to (E \otimes F)_r.$$

Since E, F are Λ_0-nuclear, the quotient maps

$$\phi_p : E \to E_p \quad \textit{and} \quad \phi_q : F \to F_q$$

are Λ_0-nuclear and so the map

$$\phi_p \otimes \phi_q : E \otimes_\pi F \to E_p \otimes_\pi F_q$$

is Λ_0-nuclear. It follows that the map

$$f = h \circ (\phi_p \otimes \phi_q) : E \otimes_\pi F \to (E \otimes F)_r$$

is Λ_0-nuclear. Since f is the canonical surjection, it follows that $E \otimes_\pi F$ is Λ_0-nuclear.

In view of Proposition 3.3, we have the following

Corollary 3.7 *Consider the following property for P :*

($\star$) *For each $\alpha \in P$ there exists $\beta \in P$ such that $\sup_n \alpha_{n^2}/\beta_n < \infty$. Then*

a) If ($\star$) holds, then (1)-(6) of the preceding Theorem hold.

b) If P is countable, then property ($\star$) is equivalent to each of the (1)-(5) in the preceding Theorem.

Proposition 3.8 *Let $\Lambda = \Lambda_{\gamma,\infty}$, where $\gamma = (\gamma_n)$ is not bounded. Then, the following are equivalent :*

(1) $\sup_n \gamma_{n^2}/\gamma_n < \infty$.

(2) If $\zeta, \eta \in \Lambda = \Lambda_0$, then there exists a bijection $\sigma = (\sigma_1, \sigma_2 : N \to N \times N$ such that $(\xi_{\sigma_1(n)}\eta_{\sigma_2(n)}) \in \Lambda_0$.

Proof : (1) $\Rightarrow$ (2) Let $d = \sup_n \gamma_{n^2}/\gamma_n$. Then $d \geq 1$. Given $\rho > 1$, let $\rho_1 = \rho^d$. Then

$$\rho^{\gamma_{n^2}}/\rho_1^{\gamma_n} \leq \rho^{d\gamma_n}/\rho_1^{\gamma_n} = 1.$$

Now the implication follows from Proposition 3.3.

(2) $\Rightarrow$ (1) If $\alpha^{(m)} = (m^{\gamma_n})$, for $m = 2, 3, \ldots$ and if $P = \{\alpha^{(m)} : m \geq 2\}$, then $\Lambda_0 = \Lambda_0(P)$. In view of proposition 3.3, for each $\alpha \in P$ there exists $\beta \in P$ such that $\sup_n \alpha_{n^2}/\beta_n < \infty$. Hence, there exists $m \geq 2$ such that $\sup_n 2^{\gamma_{n^2}}/m^{\gamma_n} < \infty$. Suppose now that $\sup_n \gamma_{n^2}/\gamma_n = \infty$. Choose indices $n_1 < n_2 < \ldots$ such that $\gamma_{n_k^2}/\gamma_{n_k} > k$. If $2^k > m$, then

$$2^{\gamma_{n_k^2}}/m^{\gamma_{n_k}} \geq (\frac{2^k}{m})^{n_k} > \frac{2^k}{m} \to \infty \quad as \quad k \to \infty,$$

a contadiction.

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