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SEPARATING MAPS AND

THE NONARCHIMEDEAN HEWITT THEOREM

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Abstract. If X and Y are zerodimensional spaces and $T : C(X) \to C(Y)$ is a *biseparating map* then the N-compactifications of X and Y are homeomorphic, provided that $\mathbf{K}$ is a commutative nontrivially valued nonarchimedean field whose residue class field has nonmeasurable cardinality. We deduce the nonarchimedean counterpart to the well-known Hewitt theorem, this is, if $C(X)$ and $C(Y)$ are ring-isomorphic, then the N-compactifications of X and Y are homeomorphic. Also, if X and Y are N-compact and T is linear and biseparating, then it is a *weighted* composition map : for some *weight function a* in $C(Y)$, $Tf = a(f \circ h)$, being $h : Y \to X$ a homeomorphism.

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The classical Gelfand-Kolmogoroff theorem states that, for separated compact spaces X and Y, if the spaces of real-valued continuous functions $C(X)$ and $C(Y)$ are ring-isomorphic, then X and Y must be homeomorphic. Afterwards it was proven ([6], [8] and [11]) that every linear separating isomorphism T from $C(X)$ onto $C(Y)$ (see below) derives in a natural way from a homeomorphism, that is, there are a homeomorphism h from Y onto X and $a \in C(Y)$, $|a(y)| \neq 0$ for every $y \in Y$, such that $Tf = a(f \circ h)$ for every $f \in C(X)$; in particular, T must be continuous.

The nonarchimedean version of this results, including as a corollary a nonarchimedean Gelfand-Kolmogoroff theorem, has been studied mainly in [12].

A generalized version of the Gelfand-Kolmogoroff theorem, due to Hewitt ([10]), asserts that, for realcompact spaces X and Y, if the spaces of real-valued continuous functions $C(X)$ and $C(Y)$ are ring-isomorphic, then X and Y must be homeomorphic. In [13], we

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proved that X and Y must be homeomorphic when there exists a biseparating map (see below) from $C(X)$ onto $C(Y)$. The aim of this paper is the study of this kind of maps in the nonarchimedean context. We give the general form of such maps when they are linear and the spaces X and Y are N-compact. In particular we prove the continuity of these maps.

Let $C(X)$, $C(Y)$ be the spaces of continuous $\mathbf{K}$-valued functions on the zerodimensional spaces X and Y respectively, where $\mathbf{K}$ is a nonarchimedean commutative complete nontrivially valued field. For $f \in C(X)$, we define the cozero set of f as $c(f) := \{x \in X : f(x) \neq 0\}$. A map T from $C(X)$ onto $C(Y)$ is said to be separating if it is bijective, additive and if $c(f) \cap c(g) = \emptyset$ implies $c(Tf) \cap c(Tg) = \emptyset$ for all $f, g \in C(X)$. T is said to be biseparating if both T and T^{-1} are separating. Important examples of biseparating maps are ring-isomorphisms, but there are many others, as we shall see.

Separating maps were introduced in the nonarchimedean context by E. Beckenstein and L. Narici ([5]) and were first used to prove the non-existence of a nonarchimedean Banach-Stone theorem in [5] and [2]. Other papers where nonarchimedean separating maps are studied are [1], [3] and [6].

In this paper we shall denote by $\beta_0 X$ and $\upsilon_0 X$ the Banaschewski compactification and the N-compactification of X respectively ([13], [4]). If $f \in C(\beta_0 X), C(\upsilon_0 X)$, we denote by $f|_X$ its restriction to X. If $f \in C(X)$, we denote by $f^{\upsilon_0 X} : \upsilon_0 X \to \mathbf{K}$ its extension to υX (when possible). Being $\omega\mathbf{K}$ any zerodimensional compactification of $\mathbf{K}$, if $f \in C(X)$, we denote by $f^{\beta_0 X} : \beta_0 X \to \omega\mathbf{K}$ the extension of f to $\beta_0 X$; when $\mathbf{K}$ is locally compact we consider $\omega\mathbf{K} = \mathbf{K} \cup \{\infty\}$ the one-point compactification of $\mathbf{K}$. For a clopen subset A of X, ξ_A will be the characteristic function of A. If $A \subset X$, then $\mathrm{cl}_{\beta_0 X} A$ stands for the closure of A in $\beta_0 X$. 1_X denotes the function identically equal to 1 on X.

Definition 1 : Let T be a separating map from $C(X)$ into $C(Y)$. A point $x_0 \in \beta_0 X$ is said to be a T-support point of $y_0 \in Y$ if, for every neighborhood $U(x_0)$ of x_0, there exists $f \in C(X)$ satisfying $c(f) \subset U(x_0)$ such that $(Tf)(y_0) \neq 0$.

Lemma 2 : *For every $y \in Y$, there exists a unique T-support point of y in $\beta_0 X$.*

This result and its proof, for the real and complex case, can be found in [8], p. 260. A similar proof can be done for the nonarchimedean case.

Lemma 3 : *Suppose that T is separating and $x_0 \in \beta_0 X$ is the T-support point of $y_0 \in Y$. If $f^{\beta_0 X}(x_0) = 0$, $f \in C(X)$, then $(Tf)(y_0) = 0$.*

Proof : Suppose that $f^{\beta_0 X}(x_0) = 0$ and $(Tf)(y_0) \neq 0$. Let (α_n) be a sequence in $\mathbf{K}$ such that $(|\alpha_n|)$ is strictly increasing and tends to infinity. Then for each $n \in \mathbf{N}$ consider the clopen subset of X,

$$U_n := \{x \in X : |f(x)| \in [1/|\alpha_{n+1}|^2, 1/|\alpha_n|^2]\},$$

and let $f_n = f\xi_{U_{2n}} \in C(X)$. Then it is not difficult to prove that the functions

$$g_1 := \sum_{n=1}^{\infty} f_n, g_2 := f - g_1$$

belong to $C(X)$. Also $g_2 = \sum_{n=1}^{\infty} h_n$ where $h_n = f\xi_{U_{2n-1}}$, for $n > 1$ and $h_1 = f\xi_V$ where

$$V = \{x \in X : |f(x)| \geq 1/2\}.$$

Since $f = g_1 + g_2$, one of these maps satisfies $(Tg_i)(y_0) = \alpha \neq 0$; without loss of generality, suppose this is the case when $i = 1$. Notice that if $n \neq m$, then $c(f_n) \cap c(f_m) = \emptyset$.

Let us see that $g := \sum_{n=1}^{\infty} Tf_n$ belongs to $C(Y)$. From the separating property of T we deduce that g is continuous at every point of $Y - \text{bdry} \bigcup_{n=1}^{\infty} c(Tf_n)$. Suppose that g is not continuous at $y \in \text{bdry} \bigcup_{n=1}^{\infty} c(Tf_n)$. Then there exists $\epsilon > 0$ such that for any neighborhood $U(y)$ of y, there is a $z \in U(y)$ such that $|g(z)| > \epsilon$. Since $g(z) \neq 0$, then there exists $n_z \in \mathbb{N}$ such that $z \in c(Tf_{n_z})$. On the other hand it is not difficult to prove that $\sum_{n=1}^{\infty} \alpha_n f_n$ belongs to $C(X)$. Therefore

$$\left| (T(\sum_{n=1}^{\infty} \alpha_n f_n))(z) \right| = \left| \alpha_{n_z} (Tf_{n_z})(z) + (T(\sum_{n \neq n_z} \alpha_n f_n))(z) \right|,$$

which is equal to $|\alpha_{n_z} g(z)|$, since T is separating. We deduce that in any neighborhood of y there are arbitrarily big values of $T(\sum_{n=1}^{\infty} \alpha_n f_n)$ and therefore $T(\sum_{n=1}^{\infty} \alpha_n f_n)$ cannot be continuous at y, which is absurd. Then we conclude that g belongs to $C(Y)$.

Next let us prove that $Tg_1 = g$. If $Tg_1 \neq g$, then $T^{-1}g = g_1 + k$ where $k \in C(X)$ is not equal to zero. Since T is separating, $c(g_1) \cap c(k) \neq \emptyset$, that is, there exists $n_0 \in \mathbb{N}$ such that $c(f_{n_0}) \cap c(k) \neq \emptyset$. Let U be a nonempty clopen subset of $c(f_{n_0}) \cap c(k)$. Then $k\xi_U \in C(X)$ and $c(k\xi_U) \cap c(f_n) = \emptyset$ for $n \neq n_0$ and, by the separating property of T,

$$c(T(k\xi_U + f_{n_0})) \cap c(T(\sum_{n \neq n_0} f_n)) = \emptyset.$$

Let $y \in Y$ be such that $(T(k\xi_U))(y) \neq 0$. Then

$$g(y) = (T(k\xi_U))(y) + (T(f_{n_0}))(y) + (T(\sum_{n \neq n_0} f_n))(y) =$$

$$(T(k\xi_U))(y) + (T(f_{n_0}))(y) \neq (T(f_{n_0}))(y) = g(y),$$

which is absurd.

Since $(Tg_1)(y_0) \neq 0$ we deduce that there exists $n_1 \in \mathbf{N}$ such that $y_0 \in c(Tf_{n_1})$. We can easily see that $x_0 \in \mathrm{cl}_{\beta_0 X} c(f_{n_1})$, that is,

$$x_0 \notin \mathrm{cl}_{\beta_0 X}\{x \in X : |f(x)| \leq 1/\left|\alpha_{n_1+2}\right|^2\},$$

which is a contradiction.

Lemma 4 : *Let T be biseparating and let $f, g \in C(X)$ be such that $c(f) \subset c(g)$. Then $c(Tf) \subset \mathrm{cl}(c(Tg))$.*

Proof : Suppose that there exists $y_0 \in c(Tf) - \mathrm{cl}(c(Tg))$. Let U be a clopen neighborhood of y_0 such that $U \cap \mathrm{cl}(c(Tg)) = \emptyset$. Then we have that $c(\xi_U) \cap c(Tg) = \emptyset$ and, since T^{-1} is separating, $c(T^{-1}\xi_U) \cap c(g) = \emptyset$; then $c(T^{-1}\xi_U) \cap c(f) = \emptyset$ and, by the separating property of T, $c(\xi_U) \cap c(Tf) = \emptyset$, which contradicts our assumption on y_0.

Proposition 5 : *Let T be biseparating. Then the T-support point of every $y \in Y$ belongs to $v_0 X$.*

Proof : Suppose $y_0 \in Y$ and that x_0, its T-support point belongs to $\beta_0 X - v_0 X$. Then there exists ([4]) a decreasing sequence (U_n) of clopen neighborhoods of x_0 such that $\bigcap_{n=1}^{\infty} U_n \cap X = \emptyset$.

For every $n \in \mathbf{N}$, let $(f_n) = (\xi_{V_n})$ where V_n is a clopen neighborhood of y_0, in such a way that, $V_n \subset V_{n-1}$ and also $c(T^{-1}f_n) \subset U_n$ for every $n \geq 2$.

Then, given (α_n) a sequence in $\mathbf{K}$ such that $(|a_n|)$ is strictly increasing and tends to infinity, the map

$$g := \sum_{n=1}^{\infty} \alpha_n T^{-1}[f_n - f_{n+1}]$$

belongs to $C(X)$, so $Tg \in C(Y)$ and $(Tg)(y_0) = \alpha \in \mathbf{K}$. Then we have that given $\epsilon > 0$ there exists a neighborhood $U(y_0)$ of y_0 such that, if $y \in U(y_0)$, then $|(Tg)(y) - \alpha| < \epsilon$.

On the other hand, we have that $y_0 \notin \mathrm{int} \bigcap_{n=1}^{\infty} V_n$ because, otherwise, there would exist $\xi_U \in C(Y)$, $\xi_U \neq 0$, satisfying $U \subset \mathrm{int} \bigcap_{n=1}^{\infty} V_n$ and then, according to Lemma 4, $c(T^{-1}\xi_U) \subset \bigcap_{n=1}^{\infty} U_n$ which obviously cannot be true.

Therefore we have that $y_0 \notin \mathrm{int} \bigcap_{n=1}^{\infty}(V_n \cap U(y_0))$, that is, for every $k \in \mathbf{N}$,

$$V_k \cap U(y_0) \not\subset \bigcap_{n=1}^{\infty}(V_n \cap U(y_0))$$

or, equivalently, there exists $y \in V_k \cap U(y_0)$ such that $y \notin V_{n_0}$ for some n_0. Thus there are $y_1 \in U(y_0)$ and $k_0 \in \mathbf{N}$ such that $y_1 \in V_{k_0}$, $y_1 \notin V_{k_0+1}$ and $|\alpha_{k_0}| > |\alpha| + \epsilon$. Then

$$(Tg)(y_1) = \alpha_{k_0}[f_{k_0}(y_1) - f_{k_0+1}(y_1)] + \alpha_{k_0+1}[f_{k_0+1}(y_1) - f_{k_0+2}(y_1)] +$$

$$[T(\sum_{n=k_0+2}^{\infty} \alpha_n T^{-1}[f_n - f_{n+1}])](y_1).$$

We have that for $n \geq k_0 + 2$,

$$c(f_{k_0} - f_{k_0+1}) \cap c(f_n - f_{n+1}) = \emptyset,$$

and since T^{-1} is separating, then

$$c(T^{-1}[f_{k_0} - f_{k_0+1}]) \cap c(T^{-1}[f_n - f_{n+1}]) = \emptyset,$$

which implies that

$$c(f_{k_0} - f_{k_0+1}) \cap c(T(\sum_{n=k_0+2}^{\infty} \alpha_n T^{-1}[f_n - f_{n+1}])) = \emptyset,$$

and therefore

$$(Tg)(y_1) = \alpha_{k_0} - \alpha_{k_0} f_{k_0+1}(y_1) + \alpha_{k_0+1} f_{k_0+1}(y_1) = \alpha_{k_0}$$

and then $|(Tg)(y_1) - \alpha| > \epsilon$, which is a contradiction.

Remember that if $f \in C(X)$ and $\mathbf{K}$ is N-compact, then f can be extended to a unique continuous map in $C(\upsilon_0 X)$ ([13], p. 42) and that $\mathbf{K}$ is N-compact if and only if and only if its residue class field has nonmeasurable cardinal ([13], p. 41]. Then we can obtain the following result.

Proposition 6 : *Suppose that the residue class field of* $\mathbf{K}$ *has nonmeasurable cardinal. If there exists a biseparating map T from $C(X)$ onto $C(Y)$, then $\upsilon_0 X$ is homeomorphic to $\upsilon_0 Y$.*

Proof : Since the residue class field of $\mathbf{K}$ has nonmeasurable cardinal, we can extend $T : C(X) \to C(Y)$ to a bijective, additive map $T^{\upsilon_0} : C(\upsilon_0 X) \to C(\upsilon_0 Y)$ by defining $T^{\upsilon_0} f := (Tf|_X)^{\upsilon_0 Y}$, $f \in C(\upsilon_0 X)$. Next we are going to see that T^{υ_0} is biseparating : if $c(f) \cap c(g) = \emptyset$, f, $g \in C(\upsilon_0 X)$, then $c(f|_X) \cap c(g|_X) = \emptyset$ and, since T is separating, $c(Tf|_X) \cap c(Tg|_X) = \emptyset$; if $c((Tf|_X)^{\upsilon_0 Y}) \cap c((Tg|_X)^{\upsilon_0 Y}) \neq \emptyset$, then its intersection with Y is not empty, this is, there exists $y_0 \in Y$ such that $(Tf|_X)(y_0) \neq 0$ and $(Tg|_X)(y_0) \neq 0$, which cannot be possible.

So we may assume that X and Y are N-compact, and we are going to prove that they are homeomorphic. Define the map $h : Y \to X$ where $h(y)$ is the T-support point of $y \in Y$. For $x_0 \in X$, consider the T^{-1}-support point $k(x_0)$ of x_0 in Y. In this way we can define a map $k : X \to Y$. Let us see that h is continuous.

Consider U a clopen subset of X containing $h(y_0)$, $y_0 \in Y$. If $y \in c(T\xi_U)$, then $h(y) \in X$, by Proposition 5, and $(\xi_U{}^{\beta_0 X}(h(y)) \neq 0$, by Lemma 3. We deduce that $h(y) \in U$

and then h is continuous. In the same way we can prove that k is continuous too. Let us see also that $h \circ k = i_X$. Let $x_0 \in X$. Consider a clopen subset U of X containing $h(k(x))$. By Lemma 3, $(T\xi_{X-U})(k(x_0)) = 0$ and applying Lemma 3 to T^{-1}, we have that $\xi_{X-U}(x_0) = 0$, this is, $x_0 \in U$. This implies $x_0 = h(k(x_0))$ and then $h \circ k = i_X$. In the same way we can prove that $k \circ h = i_Y$ and then h and k are homeomorphisms.

As a corollary, taking into account that every ring isomorphism is a biseparating map, we obtain a nonarchimedean version of the Hewitt theorem.

Corollary 6 : *Suppose that the residue class field of* $\mathbf{K}$ *has nonmeasurable cardinal. If* $C(X)$ *and* $C(Y)$ *are isomorphic as rings, then* v_0X *and* v_0Y *are homeomorphic.*

Looking carefully at the proof of Proposition 6, we can see that we make use of the measurability of the cardinal of the residue class field of $\mathbf{K}$ just to extend the functions of $C(X)$ to functions defined on v_0X. But if X and Y are N-compact, then the second part of the proof is valid to show the following result.

Corollary 7 : *Suppose that* X *and* Y *are* N*-compact. If* $C(X)$ *and* $C(Y)$ *are isomorphic as rings, then* X *and* Y *are homeomorphic.*

Proposition 8 : *If* T *is a linear biseparating map from* $C(X)$ *onto* $C(Y)$ *and* X, Y *are* N*-compact spaces, then there exist* $a \in C(Y)$, $a(y) \neq 0$ *for all* $y \in Y$, *and a homeomorphism* $h : Y \to X$ *such that* $(Tf)(y) = a(y)f(h(y))$ *for every* $f \in C(X)$, $y \in Y$. *In particular, if* $C(X)$ *and* $C(Y)$ *are endowed with the compact-open topology, then* T *is continuous.*

Proof : By Lemma 3, we have that if $(Tf)(y) \neq 0$, then $f(h(y)) \neq 0$, and, applying Lemma 3 to T^{-1}, if $f(x) \neq 0$, then $(Tf)(k(x)) \neq 0$. Since k is a homeomorphism, we have that

$$a(y) := (T1_X)(y) \neq 0$$

for every $y \in Y$. We know that $(Tf)(y) \neq 0$ if and only if $f(h(y)) \neq 0$, so if $(Tf)(y) = 0$, then $(Tf)(y) = a(y)f(h(y))$. Suppose that $(Tf)(y) \neq 0$. Let $\alpha \in \mathbf{K}$ be such that $\alpha(Tf)(y_0) + a(y_0) = 0$; then we have that $\alpha f(h(y_0)) + 1 = 0$, which implies that

$$\alpha a(y_0)f(h(y_0)) + a(y_0) = 0.$$

Then $(Tf)(y_0) = a(y_0)f(h(y_0))$.

Let us next show that T is continuous. Take a compact subset K of Y. Given $\epsilon > 0$, if for $x \in h(K)$, $|f(x)| < \epsilon/\sup\{y \in K : |a(y)|\}$, it is easy to check that $|(Tf)(y)| < \epsilon$, for every $y \in K$.

Remarks : 1. In general, if X and Y are not N-compact, a biseparating linear map need not be continuous. As an example of this fact, consider X a pseudocompact not N-compact space and Y its N-compactification, which is compact ([14], Corollary 1.6). Suppose that $\mathbf{K}$ is N-compact. It is easy to see that the canonical isomorphism $T : C(X) \to C(Y)$, defined as $Tf := f^{\upsilon_0 X}$ is well defined (since $\mathbf{K}$ is $\mathbf{N}$-compact), biseparating, but not continuous : given $\epsilon > 0$, there is no compact subset K of X such that if $|f(x)| < \epsilon$ for every $x \in K$, then $|f^{\upsilon_0 X}(x)| < \epsilon$ for every $x \in \upsilon_0 X = \beta_0 X$.

2. If we do not assume T to be linear, then the map $a \in C(Y)$ of Proposition 8 may not exist. Suppose that $\mathbf{K} = \mathbf{Q}_p$. Consider $X = Y = \{x\}$. Let $\mathbf{r}$ stand for the map $f(x) = r$, $r \in \mathbf{Q}_p$. Take $s \in \mathbf{Q}_p - \mathbf{Q}$. Consider $\{\mathbf{1}, \mathbf{s}\} \cup \{h_i : i \in I\}$ a basis of $C(X)$ as a vector space over $\mathbf{Q}$. Define $T\mathbf{1} := \mathbf{s}, T\mathbf{s} := \mathbf{1}, Th_i := h_i, i \in I$, and extend T by linearity (over $\mathbf{Q}$) to the whole space $C(X)$.

Then $T : C(X) \to C(X)$ is a biseparating map. If $(Tf)(x) = a(x)f(x)$ for every $f \in C(X)$, then taking $f = \mathbf{1}$, $a = \mathbf{s}$, and taking $f = h_i$, $a = \mathbf{1}$, which is not possible.

Proposition 9 : *Suppose that* $\mathbf{K}$ *is locally compact. If* X *is zerodimensional and* T *is linear and separating, then* T *is biseparating.*

Proof : If T was not separating, there would exist two maps f and $g \in C(X)$ such that $c(f) \cap c(g) \neq \emptyset$. Take U a nonempty clopen subset of $c(f) \cap c(g)$ such that f and g are bounded on U and there exists $\alpha \in \mathbf{R}$, $\alpha > 0$ such that $|f(x)| \geq \alpha$ and $|g(x)| \geq \alpha$ for every $x \in U$. By putting $f = f\xi_U + f\xi_{X-U}$ and $g = g\xi_U + g\xi_{X-U}$, it is easy to see that

$$c(T(f\xi_U)) \cap c(T(g\xi_U)) = \emptyset.$$

Suppose that $y_0 \in Y$ is such that $(T(f\xi_U))(y_0) \neq 0$. Let $x_0 \in \beta_0 X$ the T-support point of y_0. Let $\gamma \neq 0$ be such that

$$(\gamma(f\xi_U)^{\beta_0 X} + (g\xi_U)^{\beta_0 X})(x_0) = 0.$$

Then $(\gamma f\xi_U + g\xi_U)^{\beta_0 X}(x_0) = 0$; therefore

$$(T(\gamma f\xi_U + g\xi_U))(y_0) \neq 0,$$

in contradiction with Lemma 3.

From Propositions 8 and 9, we deduce the following corollary.

Corollary 10 : *Let* $\mathbf{K}$ *be locally compact. If* T *is a linear separating map from* $C(X)$ *onto* $C(Y)$ *and* X, Y *are* $\mathbf{N}$*-compact spaces, then there exist* $a \in C(Y)$, $a(y) \neq 0$ *for all* $y \in Y$, *and a homeomorphism* $h : Y \to X$ *such that* $(Tf)(y) = a(y)f(h(y))$ *for every* $f \in C(X)$, $y \in Y$. *In particular, if* $C(X)$ *and* $C(Y)$ *are endowed with the compact-open topology, then* T *is continuous.*

Remark : Note that in Proposition 9 and Corollary 10, if X is compact, we can get the same results, with the same proofs, even if $\mathbf{K}$ is not locally compact.

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